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35/39 + 1

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2.1 Solving for Exponents

$$a^n \times a^m = a^{m+n} \qquad a^n \div a^m = \frac{a^n}{a^m} = a^{n-m} \qquad \left(a^n\right)^m = a^{m \times n}$$

1. Find the missing value in the box for each of the following equations:

a) $3^{\boxed{4}} = 81$ ✓

b) $4^{\boxed{3}} = 64$ ✓

c) $9^{\boxed{3}} = 729$ ✓

d) $8^{\boxed{\frac{16}{3}}} = 65536$ ✓

e) $5^{\boxed{7}} = 78125$ ✓

f) $6^{\boxed{5}} = 7776$ ✓

2. Simplify each of the following expressions in exponential form:

a) $8 \times 2^5 = \boxed{2^8}$ ✓ ($8 = 2^3$)

b) $\frac{27}{3^2} = \boxed{3^1}$ ✓ ($27 = 3^3$)

c) $\left(\frac{512}{2^5}\right)^2 = \boxed{2^8}$ ✓ ($512 = 2^9$, $(2^4)^2$)

3. Solve for the unknown variable "x" in each of the following expressions:

a) $3^x = 81$

$x = 4$ ✓

b) $2^x = 256$

$x = 8$ ✓

c) $12^x = 2^6 3^3$ ($= 4^3 3^3$)

$x = 3$ ✓

d) $243 = \frac{3^x}{27}$ (3^5, 3^3)

$x = 8$ ✓

e) $\frac{64}{2^x} = 1024$ (2^6, 2^{10})

$x = -4$ ✓

f) $(27)^3 = 3^x$ ($(3^3)^3 = 3^9$)

$x = 9$ ✓

4. Find the value of variable in each of the following equations:

a) $3^{2x} = 729$ (3^6)

$2x = 6$

$x = 3$ ✓

b) $4^{3x} = 128$ (2^7)

$2^{2(3x)} = 2^{6x} = 2^7$

$6x = 7$

$x = \frac{7}{6}$ ✓

c) $729^x = 81$ (9^2, 9^{3x})

$3x = 2$

$x = \frac{2}{3}$ ✓

d) $4^{x+1} = 65536$

$4^x = \frac{65536}{4} = 16384$

$= 4^7$

$x = 7$ ✓

e) $25^{2x} = 78125 = 5^7$

$5^{4x} = 5^7 \quad 4x = 7$

$x = \frac{7}{4}$ ✓

f) $36^{x+3} = 7776 = 6^5$

$6^{2(x+3)} = 6^5$

$2x + 6 = 5$

$2x = -1$

$x = -\frac{1}{2}$ ✓

5. Find the value of variable in each of the following equations:

$4^x(16) = 64 = 4^3$ ($16 = 4^2$)

$4^{x+2} = 4^3 \quad x + 2 = 3$

$x = 1$ ✓

b) $\left(3^{2x}\right)^3 = 81 = 3^4$

$3^{6x} = 3^4 \quad 6x = 4$

$x = \frac{4}{6} = \frac{2}{3}$ ✓

$x = \frac{2}{3}$

c) $\frac{16^x}{2^{3x}} = 128 = 2^7$ ($16 = 2^4$)

$\frac{2^{4x}}{2^{3x}} = 2^x = 2^7$

$x = 7$ ✓

d) $\left(4^{x+1}\right)\left(2^5\right) = 65536$

$= 4^8 = 2^{16}$

$2^{2x+2+5} = 2^{16}$

$2x + 7 = 16 \quad 2x = 9$

$x = \frac{9}{2}$ ✓

e) $\frac{\left(729^x\right)}{9^{x-3}} = 3^1$ ($729^x = 3^{6x}$, $9^{x-3} = 3^{2(x-3)}$)

$3^{6x-(2x-6)} = 3^{6x-2x+6}$

$4x + 6 = 1 \quad 4x = -5$

$x = -\frac{5}{4}$ ✓

f) $\left(\frac{8^{-1} + 2^{-3}}{4^{-3}}\right)^3 = 32^{-x+1} = 2^{5(-x+1)}$

$\left(\frac{2^{-3} + 2^{-3}}{2^{-6}}\right)^3 = 2^{-5x+5}$

$\left(\frac{2^{-6}}{2^{-6}}\right)^3 = \frac{2^{-18}}{2^{-18}} = 2^{-5x+5}$

$x = 1$ ✗

$-18 = -18 + -5x + 5$

$-18 = -13 + -5x$

$5x = 5$

6. Given the equation, find the smallest value of x+y:

a. $12^4 = 2^x 3^y$

$4^4 \cdot 3^4 = 2^8 \cdot 3^4$

$8 + 4 = 12$ ✓

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b) $3969^3 = x^6 9^y$

$3969^3 = (7^2 9^2)^3$

$x = 7 \quad y = 6$ ✓

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c) $x^y = y^x \quad (x \neq y)$

$2^4 = 4^2$

$2 + 4 = 6$ ✓

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7. Solve for "x" in each of the following equations:

a) $\left(\frac{81^{x+4}}{9^5}\right)^4 = \left(\frac{1}{3}\right)^x$

$\left(\frac{3^{4(x+4)}}{3^{5(5)}}\right)^4 = \frac{3^{4(4x+16)}}{3^{5(15)}} = \frac{1}{3^x}$

$3^{16x+64+x} = 3^{75}$

$17x + 64 = 75$

$17x = 11$

$x = \frac{11}{17}$

$= -\frac{24}{17}$ ✗

b) $\left(\frac{16^{-x}}{32^2}\right)^4 = 64^{x+1}$

$\left(\frac{2^{-4x}}{2^{10}}\right)^4 = 2^{6x+6}$

$2^{-16x} = 2^{6x+6+40}$

$-16x = 6x + 46$

$-22x = 46$

$x = -\frac{46}{22}$

$= -\frac{23}{11}$

$x = -\frac{23}{11}$ ✓

c) $\left(\frac{\left(81^2\right)9^x}{729^3}\right)^{-1} = \frac{\left(3^{3x+1}\right)}{9^x}$

$\left(\frac{3^8\, 3^{2x}}{3^{18}}\right)^{-1} = \frac{3^{3x+1}}{3^{2x}}$

$3^{-2x-8+2x} = 3^{3x+1+18}$

$3^{-8} = 3^{3x+19}$

$3x = 19 + 8 = 27$

$x = 9$

$x = 9$ ✗

$x = 3$

8. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then what is the value of xyz?

7^3

$x = \frac{7^5}{y^2}$ $y = y$ $z^3 = \frac{7^4}{x^2y} = \frac{7^4}{\left(\frac{7^5}{y^2}\right)y} = \frac{7^4}{\frac{7^{10}}{y^4}y} = 7^4 \cdot \frac{y^3}{7^{10}}$ $z = \sqrt[3]{7^4 \cdot \frac{y^3}{7^{10}}} = \sqrt[3]{\frac{y^3}{7^6}} = \frac{y}{7^2}$

$xyz = \frac{7^5}{y^2} \cdot y \cdot \frac{y}{7^2} = \frac{7^5 y^2}{y^2 7^2} = 7^3$

9. Solve for "x": $2\left(2^{2x}\right) = 4^x + 64$.

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$2(4^x) = 4^x + 64$

$4^x + 4^x = 4^x + 64$

$64 = 4^x = 4^3$

$x = 3$

10. Let "a" and "b" be real numbers, with $a > 1$ and $b > 0$. If $ab = a^b$ and $\frac{a}{b} = a^{3b}$, determine the value of 'a"

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$ab = a^b$

$\frac{a}{b} = a^{3b}$ $\frac{a}{b} = (a^b)^3 = (ab)^3$

$\frac{a}{b} = a^3b^3$ $\frac{1}{b} = a^2b^3$

$a^2 = \frac{1}{b^4}$ $a = \frac{1}{b^2}$

$\frac{a}{b} = \frac{\frac{1}{b^2}}{b} = \left(\frac{1}{b^2}\right)^{3b}$

$\frac{1}{b^3} = \left(\frac{1}{b^6}\right)^b$ $\frac{1}{b^3} = \frac{1}{b^{6b}}$ $b^3 = (b^6)^b$ $b = \frac{1}{2}$

$\therefore a = \frac{1}{b^2} \Rightarrow a = \frac{1}{\left(\frac{1}{2}\right)^2} = \frac{1}{\frac{1}{4}} = 4$

11. Challenge: If $a = 3^p$, $b = 3^q$, $c = 3^r$, *and* $d = 3^s$ and if *p,q,r, and s* are positive integers, determine the smallest value of $p+q+r+s$ such that: $a^2 + b^3 + c^5 = d^7$

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$3^{2p} + 3^{3q} + 3^{5r} = 3^{7s}$

$3^{90} + 3^{90} + 3^{90} = 3^{91}$

30	31
60	61
90	91
120	121

$p = 45$ $q = 30$

$r = 18$ $s = 13$ } 106

12. If x and y are integers with $(y-1)^{x+y} = 4^3$, then what is the number of possible values for "x"

(A) 8 (B) 3 (C) 4 (D) 5 (E) 6

$2^6, 4^3, 8^2, 64^1, (-2)^6, (-8)^2$

6 possibilities

13. Suppose $N = 1+11+101+1001+\ldots\ldots+1000\ldots\ldots000001$ (last term has 50 zeroes) When "N" is calculated, and written as a single integer, what is the sum of its digits?

(A) 50 (B) 99 (C) 55 (D) 58 (E) 103

51
11
101
1001
} 50
62
50

SUM: $50 + 6 + 2 = 58$

$$\left(\frac{81^{x+4}}{9^{5}}\right)^{4} = \left(\frac{1}{3}\right)^{x}$$

$$= \left(\frac{3^{4x+16}}{3^{10}}\right)^{4} = \left(3^{-x}\right)$$

$$\left(3^{4x+6}\right)^{4} = 3^{-x}$$

$$3^{16x+24} = 3^{-x}$$

$$16x + 24 = -x$$

$$x + 16x = -24$$

$$17x = -24$$

$$x = \frac{-24}{17}$$

$$\left(\frac{(81^{2})9^{x}}{(729)^{3}}\right)^{-1} = \frac{3^{3x+1}}{9^{x}}$$

$$\left(\frac{3^{8}\,3^{2x}}{3^{18}}\right)^{-1} = \frac{3^{3x+1}}{3^{2x}}$$

$$\left(3^{8+2x-18}\right)^{-1} = 3^{x+1}$$

$$3^{10-2x} = 3^{x+1}$$

$$3x = 9 \qquad x = 3$$

$2^1 = 2$	$3^1 = 3$	$5^1 = 5$	$6^1 = 6$
$2^2 = 4$	$3^2 = 9$	$5^2 = 25$	$6^2 = 36$
$2^3 = 8$	$3^3 = 27$	$5^3 = 125$	$6^3 = 216$
$2^4 = 16$	$3^4 = 81$	$5^4 = 625$	$6^4 = 1296$
$2^5 = 32$	$3^5 = 243$	$5^5 = 3125$	$6^5 = 7776$
$2^6 = 64$	$3^6 = 729$	$5^6 = 15625$	$6^6 = 46656$
$2^7 = 128$	$3^7 = 2187$	$5^7 = 78125$	$6^7 = 279936$
$2^8 = 256$	$3^8 = 6561$	$5^8 = 390625$	
$2^9 = 512$	$3^9 = 19683$	$5^9 = 1953125$	
$2^{10} = 1024$	$3^{10} = 59049$	$5^{10} = 9765625$	
$2^{11} = 2048$			

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MATH ENRICHED

SECTION 2.1 CORRECTIONS

5. f) $\left(\frac{8^{-1}+2^{-3}}{4^{-3}}\right)^3 = 32^{-x+1}$

↳ $\left(\frac{\frac{1}{2^3}+\frac{1}{2^3}}{\frac{1}{2^6}}\right)^3 = 32^{-x+1}$

↳ $\left(\frac{2}{2^3}\cdot\frac{2^6}{1}\right)^3 = 2^{5(-x+1)}$

↳ $\left(\frac{2^7}{2^3}\right)^3 = 2^{-5x-5}$

↳ $(2^4)^3 = 2^{-5x-5}$

↳ $12 = -5x - 5$

↳ $5x + 5 = -12$

↳ $5x = -7$

↳ $\boxed{x = -\frac{7}{5}}$

7. a) $\left(\frac{81^{x+4}}{9^5}\right)^4 = \left(\frac{1}{3}\right)^x$

↳ $\left(\frac{3^{4(x+4)}}{3^{10}}\right)^4 = \left(\frac{1}{3}\right)^x$

↳ $\left(\frac{3^{4x+16}}{3^{10}}\right)^4 = 3^{-x}$

↳ $\left(3^{4x+6}\right)^4 = 3^{-x}$

↳ $3^{16x+24} = 3^{-x}$

↳ $16x + 24 = -x$

↳ $17x = -24$

↳ $\boxed{x = -\frac{24}{17}}$

7. c) $\left(\frac{(81^2)9^x}{729^3}\right)^{-1} = \frac{(3^{3x+1})}{9^x}$

↳ $\left(\frac{3^8\cdot 3^{2x}}{3^{18}}\right)^{-1} = \frac{(3^{3x+1})}{3^{2x}}$

↳ $\frac{3^{18}}{3^{2x+8}} = \frac{3^{3x+1}}{3^{2x}}$

↳ $2x + 18 = 3x + 1 + 2x + 8$

↳ $3x = 9$

↳ $\boxed{x = 3}$